Non-Gaussian Effects on Domain Growth

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Abstract

The vacuum two-point correlation function is calculated beyond the Gaussian approximation during the second order phase transition. It is found that the correlation function is dominated by the Gaussian term immediately after the phase transition but later is taken over by non-Gaussian terms as the spinodal instability continues. The non-Gaussian effects lead to larger size domains and may imply a smaller density of topological defects than that predicted by the Hartree-Fock approximation.

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The phenomenon of continuous phase transitions is common to many diverse areas of physics. In particular phase transitions have been studied extensively in condensed matter. However the dynamical process of the onset of phase transition and temperature dependence or more generally its time-dependence is less well-understood. Topological defects may be formed during spontaneous symmetry breaking phase transitions. A theoretical understanding for defect formation in such phase transitions was provided by Kibble [1] and Zurek [2] on the basis of causality principle.

Since then, it is argued that the formation of vortices may be observed experimentally in liquid ^{4}He [3] and ^{3}He [4]. It is well known that liquid ^{4}He has a λ-point phase transition to a condensed phase, superfluid ^{4}He, at T = 2.17 K. For liquid ^{3}He the phase transition to a superfluid state occurs at 2 mK. The transition in ^{3}He is more like the case of superconductivity in metals where pairs of electrons are coupled to form loosely bound bosons in ^1S state, except that now ^{3}He is a fermion and the pairs are in ^3P state. However it appears that experiments do not give a definite result on the vortex formation.

The early Universe might have undergone a sequence of spontaneous symmetry breaking phase transitions [1,5]. Heavy-ion collisions at relativistic energies lead to a quark-gluon plasma, which is believed to mimick the early stage of the Universe. In the process of cooling this plasma, the system will go through a chiral phase transition. It is likely that the phase transition may lead to the formation of disoriented chiral condensate (DCC) [6]. The evidence of DCC would be emission of large number of coherent pions from the decay of such condensates.

Current methods to study the dynamical formation of defects from the onset of phase transitions are classified largely into the time-dependent Landau-Ginzburg (TDLG) equation [7] and quantum field theory (QFT) [8–12]. In the former the dynamics of phase transitions is described by the classical TDLG equation with a stochastic noise, whereas in the latter the dynamics is described by the quantum field equation with or without the noise. QFT is more fundamental than TDLG equation in the sense that QFT is the best tested, microscopic theory of nature. But QFT has been limited to, more or less, the Hartree-Fock or Gaussian approximation due to technical difficulty. However, during the second order phase transition via a quench, the soft (long wavelength) modes grow exponentially. So it is likely that nonlinear terms would become comparable to the Gaussian contribution during the phase transition and spinodal instability quickly turns the dynamics of the problem to be highly nonlinear and nonperturbative. It may then be anticipated that large non-Gaussian fluctuations will drive the domain size.

In this Letter we find the nonequilibrium quantum evolution beyond the Gaussian approximation during the second order phase transition and calculate the correlation length from the two-point function. As a QFT model we shall focus on a real scalar field undergoing an instantaneous quench that describes the dynamical formation of domains. For that purpose we use the Fock basis obtained in Refs. [12–14] to find perturbatively and systematically the improved vacuum state during the quench of the second order phase transition. Our scheme is more systematic than an improved Gaussian state corrected by the second excited state [15], since we can find all the corrections to the Gaussian state to any desired order of the coupling constant using the standard perturbation method.

The QFT model for the second order phase transition is described by the Hamiltonian

$$H(t) = \int d^3\mathbf{x}\left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{m^2(t)}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right], \tag{1}$$

where

$$m^2(t) = \begin{cases} m_i^2, & t < 0, \\ -m_f^2, & t > 0. \end{cases} \tag{2}$$

The quantum dynamics of nonequilibrium process is prescribed by the time-dependent functional Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{H}(t)|\Psi(t)\rangle. \tag{3}$$

By redefining the Hermitian Fourier modes as

$$\phi_{\mathbf{k}}^{(+)}(t) = \frac{1}{2}[\phi_{\mathbf{k}}(t) + \phi_{-\mathbf{k}}(t)], \quad \phi_{\mathbf{k}}^{(-)}(t) = \frac{i}{2}[\phi_{\mathbf{k}}(t) - \phi_{-\mathbf{k}}(t)], \tag{4}$$

where

$$\phi(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\phi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{5}$$

we rewrite the Hamiltonian (1) as

$$H(t) = \sum_\alpha \frac{1}{2}\pi_\alpha^2 + \frac{1}{2}\omega_\alpha^2(t)\phi_\alpha^2 + \frac{\lambda}{4!}\left[\sum_\alpha \phi_\alpha^4 + 3\sum_{\alpha\neq\beta}\phi_\alpha^2\phi_\beta^2\right], \tag{6}$$

where

$$\omega_\alpha^2(t) = m^2(t) + \mathbf{k}^2, \tag{7}$$

with α and β denoting $\{\mathbf{k}, (\pm)\}$. In deriving Eq. (6) we neglect the odd power terms from the self-interaction whose expectation values vanish with respect to symmetric states because we are interested in improving the Gaussian state that preserves the parity. The mode-decomposed Hamiltonian (6) consists of coupled quartic oscillators, the coupling among different modes arising from the last term. Because of the time-dependent mass squared the Hamiltonian (6) describes a system in nonequilibrium, the degree of nonequilibrium being determined by the rate of change of the mass. In order to handle this nonequilibrium system we use a recently introduced canonical method, the so-called Liouville-von Neumann approach [12–14].

The essential idea of Refs. [12–14] is to introduce the following time-dependent annihilation and creation operators for each mode that satisfy the quantum Liouville-von Neumann equation:

$$\hat{A}_\alpha(t) = \frac{i}{\sqrt{\hbar}}[\varphi_\alpha^*(t)\hat{\pi}_\alpha - \dot{\varphi}_\alpha^*(t)\hat{\phi}_\alpha], \quad \hat{A}_\alpha^\dagger(t) = -\frac{i}{\sqrt{\hbar}}[\varphi_\alpha(t)\hat{\pi}_\alpha - \dot{\varphi}_\alpha(t)\hat{\phi}_\alpha]. \tag{8}$$

Then the Hamiltonian (6) can be represented in terms of $\hat{A}_\alpha, \hat{A}_\alpha^\dagger$ and divided into

$$\hat{H}(t) = \hat{H}_{\rm G}(t) + (\lambda\hbar^2)\hat{H}_{\rm P}(t), \tag{9}$$

where $\hat{H}_{\rm G}$ is the quadratic (Gaussian) part and $\hat{H}_{\rm P}$ is the quartic part:

$$\begin{aligned} \hat{H}_{\rm G} = \frac{\hbar}{2}\sum_\alpha \Bigl[(\dot{\varphi}_\alpha^2 + \omega_\alpha^2\varphi_\alpha^2)\hat{A}_\alpha^2 + 2(\dot{\varphi}_\alpha^*\dot{\varphi}_\alpha + \omega_\alpha^2\varphi_\alpha^*\varphi_\alpha)\hat{A}_\alpha^\dagger\hat{A}_\alpha + (\dot{\varphi}_\alpha^{*2} + \omega_\alpha^2\varphi_\alpha^{*2})\hat{A}_\alpha^{\dagger 2}\Bigr] \\ + \frac{\lambda\hbar^2}{4}\Bigl(\sum_\beta \varphi_\beta^*\varphi_\beta\Bigr)\sum_\alpha\Bigl[\varphi_\alpha^2\hat{A}_\alpha^2 + 2\varphi_\alpha^*\varphi_\alpha\hat{A}_\alpha^\dagger\hat{A}_\alpha + \varphi_\alpha^{*2}\hat{A}_\alpha^{\dagger 2}\Bigr], \end{aligned} \tag{10}$$

and

$$\begin{aligned} \hat{H}_{\rm P} = \frac{1}{4!}\Biggl\{\sum_\alpha\sum_{k=0}^4 \binom{4}{k}\varphi_\alpha^{*(4-k)}\varphi_\alpha^k\hat{A}_\alpha^{\dagger(4-k)}\hat{A}_\alpha^k \\ +3\sum_{\alpha\neq\beta}\Bigl(\varphi_\alpha^2\hat{A}_\alpha^2 + 2\varphi_\alpha^*\varphi_\alpha\hat{A}_\alpha^\dagger\hat{A}_\alpha + \varphi_\alpha^{*2}\hat{A}_\alpha^{\dagger 2}\Bigr)\Bigl(\varphi_\beta^2\hat{A}_\beta^2 + 2\varphi_\beta^*\varphi_\beta\hat{A}_\beta^\dagger\hat{A}_\beta + \varphi_\beta^{*2}\hat{A}_\beta^{\dagger 2}\Bigr)\Biggr\}. \end{aligned} \tag{11}$$

Here we neglect purely c-number terms. The requirement that each $\hat{A}_\alpha$ and $\hat{A}_\alpha^\dagger$ satisfy the quantum Liouville-von Neumann equation

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\hat{A}_\alpha(t) + [\hat{A}_\alpha(t), \hat{H}_{\rm G}(t)] = 0, \\ i\hbar\frac{\partial}{\partial t}\hat{A}_\alpha^\dagger(t) + [\hat{A}_\alpha^\dagger(t), \hat{H}_{\rm G}(t)] = 0, \end{aligned} \tag{12}$$

leads to the mean field equation for each mode

$$\ddot{\varphi}_\alpha(t) + \omega_\alpha^2(t)\varphi_\alpha(t) + \frac{\lambda\hbar}{2}\Bigl(\sum_\beta \varphi_\beta^*(t)\varphi_\beta(t)\Bigr)\varphi_\alpha(t) = 0. \tag{13}$$

Further the standard commutation relations $[\hat{A}_\alpha(t), \hat{A}_\beta^\dagger(t)] = \delta_{\alpha,\beta}$ are guaranteed for all times by the Wronskian conditions

$$\dot{\varphi}_\alpha^*(t)\varphi_\alpha(t) - \dot{\varphi}_\alpha(t)\varphi_\alpha^*(t) = i. \tag{14}$$

Then the Fock space for each mode consists of the Gaussian vacuum and excited states with quantum number n_α

$$\hat{A}_\alpha(t)|0_\alpha, t\rangle_{\rm G} = 0, \quad |n_\alpha, t\rangle_{\rm G} = \frac{\hat{A}_\alpha^{\dagger n_\alpha}(t)}{\sqrt{n_\alpha!}}|0_\alpha, t\rangle_{\rm G}. \tag{15}$$

The Fock space basis satisfies the orthonormality condition ${}_{\rm G}\langle m_\alpha, t|n_\beta, t\rangle_{\rm G} = \delta_{\alpha\beta}\delta_{mn}$. Up to a time-dependent phase factor each state $|n_\alpha, t\rangle_{\rm G}$ satisfies the Schrödinger equation for the corresponding mode of $\hat{H}_{\rm G}$. The wave functional for $\hat{H}_{\rm G}$ is a product of such a state for each mode. For instance, the exact vacuum state $|0, t\rangle_{\rm G} = \prod_\alpha |0_\alpha, t\rangle_{\rm G}$ for $\hat{H}_{\rm G}(t)$ is the Gaussian vacuum for the total Hamiltonian $\hat{H}(t)$ in the Hartree-Fock approximation. Other excited states of the Gaussian vacuum are the multiparticle states of modes, which are compactly denoted as

$$|\{\mathcal{N}\},t\rangle_{\mathrm{G}} = \prod_{\{\mathcal{N}\}} \frac{\hat{A}^{\dagger\{\mathcal{N}\}}(t)}{\sqrt{\{\mathcal{N}\}!}}|0,t\rangle_{\mathrm{G}} \tag{16}$$

where $\{\mathcal{N}\} = (n_1, \cdots, n_\alpha, \cdots)$ with all n_α being nonnegative integers, $\hat{A}^{\dagger\{\mathcal{N}\}} = \hat{A}_1^{\dagger n_1} \cdots \hat{A}_\alpha^{\dagger n_\alpha} \cdots$, and $\{\mathcal{N}\}! = \prod_\alpha n_\alpha!$.

Quantum states beyond the Gaussian approximation can be obtained by treating $\hat{H}_{\mathrm{P}}$ as a perturbation to $\hat{H}_{\mathrm{G}}$. The perturbation $\hat{H}_{\mathrm{P}}$ excites and de-excites in pairs the multiparticle states, so we look for an exact quantum state of the form

$$|\{\mathcal{N}\},t\rangle = \hat{U}[\hat{A}_\alpha^\dagger(t), \hat{A}_\alpha(t); t, \lambda]|\{\mathcal{N}\},t\rangle_{\mathrm{G}}. \tag{17}$$

Then the Schrödinger equation (3) takes the form

$$\left[i\hbar\frac{\partial}{\partial t}\hat{U}(t,\lambda) + [\hat{U}(t,\lambda), \hat{H}_{\mathrm{G}}(t)] - \lambda\hat{H}_{\mathrm{P}}(t)\right]\prod_\alpha |\{\mathcal{N}\},t\rangle_{\mathrm{G}} = 0. \tag{18}$$

Using Eq. (12) and technically assuming that the time derivative does not act on $\hat{A}_\alpha^\dagger(t)$ and $\hat{A}_\alpha(t)$, the operator $\hat{U}$ satisfies an interaction picture-like equation

$$i\hbar\frac{\partial}{\partial t}\hat{U}(t,\lambda) = (\lambda\hbar^2)\hat{H}_{\mathrm{P}}(t)\hat{U}(t,\lambda). \tag{19}$$

The formal solution to Eq. (19) can be written as

$$\hat{U}(t,\lambda) = \mathrm{T}\exp\left[-\lambda\hbar\int \hat{H}_{\mathrm{P}}(t)dt\right], \tag{20}$$

where T denotes a time-ordered integral and $\hat{A}_\alpha^\dagger(t)$ and $\hat{A}_\alpha(t)$ are treated as if they are constant operators. The operator $\hat{U}$ is a unitary operator, which is a consequence of the Hermitian $\hat{H}_{\mathrm{p}}$ and is easily shown from the formal solution (20). The unitarity of $\hat{U}$ preserves the orthonormality of $\langle\{\mathcal{M}\},t|\{\mathcal{N}\},t\rangle = \delta_{\{\mathcal{M}\},\{\mathcal{N}\}}$. The vacuum state is then given by

$$|0,t\rangle = \hat{U}(t,\lambda)|0,t\rangle_{\mathrm{G}} \equiv \sum_{\{\mathcal{N}\}} U_{\{\mathcal{N}\}}(t)|\{\mathcal{N}\},t\rangle_{\mathrm{G}}. \tag{21}$$

Our vacuum state (21) is non-Gaussian in that it is a superposition of the Gaussian vacuum and its excited states [15], and further has kurtosis (higher moments) different from the Gaussian vacuum [16].

Before the quench ($t < 0$), the Gaussian vacuum for each mode is given by the solution

$$\varphi_{i,\mathbf{k}}(t) = \frac{1}{\sqrt{2\Omega_{i,\mathbf{k}}}}e^{-i\Omega_{i,\mathbf{k}}t}, \tag{22}$$

where Eq. (13) leads to the gap equation and yields the correct Gaussian vacuum energy [17]. The contribution of non-Gaussian terms to the correlation function is of the order of $(\lambda\hbar)^2/(2^{11}\Omega_{i,\mathbf{k}}^4)$. Thus not only for the weak coupling limit ($\lambda \ll m_i^2$) but also for the strong coupling limit $\lambda \simeq m_i^2$, the non-Gaussian contribution to the two-point function is quite negligible. This justifies the validity of the Gaussian vacuum before the phase transition via

the quench. However, after the quench ($t > 0$), the soft modes ($\mathbf{k}^2 < m_f^2$) begin to grow exponentially due to $m^2(t) = -m_f^2$. The exponential growth continues until it reaches the spinodal line where $(\lambda\hbar)(\varphi^*_{f,\mathbf{k}}\varphi_{f,\mathbf{k}})$ from the self-interaction takes over the unstable $-m_f^2$. Immediately after the quench we find approximately the soft mode solutions to Eq. (13) as

$$\varphi_{f,\mathbf{k}}(t) \approx \frac{1}{2\sqrt{2\Omega_{i,\mathbf{k}}}}\Bigg[\Bigg(1 - i\frac{\Omega_{i,\mathbf{k}}}{\tilde{\Omega}_{f,\mathbf{k}}}\Bigg)e^{\tilde{\Omega}_{f,\mathbf{k}}t} + \Bigg(1 + i\frac{\Omega_{i,\mathbf{k}}}{\tilde{\Omega}_{f,\mathbf{k}}}\Bigg)e^{-\tilde{\Omega}_{f,\mathbf{k}}t}\Bigg], \tag{23}$$

where $\tilde{\Omega}_{f,\mathbf{k}} \approx \sqrt{m_f^2 - \mathbf{k}^2}$. These soft mode solutions after the quench continuously match those before the quench and become exact for the non-interacting case ($\lambda = 0$).

However, during spinodal instability the perturbation $(\lambda\hbar)\hat{H}_{\mathrm{P}}$ grows exponentially, becomes comparable to the self-interaction term of $\hat{H}_{\mathrm{G}}$ and will make a contribution to the correlation function as much as the Gaussian contribution. Since the soft mode solution (23), which dominates over hard (short wavelength) mode solutions, has an exponentially growing factor overall time-dependent factor, $\hat{H}_{\mathrm{P}}(t)$ approximately commutes each other for two different times $t' \neq t$. Hence the formal solution (20) has a good approximation

$$\hat{U}_{[1]}(t, \lambda) = \exp\Bigg[-i\lambda\hbar\int \hat{H}_{\mathrm{P}}(t)dt\Bigg]. \tag{24}$$

Moreover, the higher order terms in an exponential form of the formal solution (see Appendices of Ref. [18]) are suppressed by the expansion parameter $\lambda\hbar$, which consolidates the validity of the operator (24) as far as soft modes are concerned. Using Eq. (24) and finding the most dominant contribution to the two-point correlation function up to order $(\lambda\hbar)^2$, we obtain

$$\begin{aligned}\langle 0, t|\hat{\phi}(\mathbf{x}, t)\hat{\phi}(\mathbf{0}, t)|0, t\rangle &= \sum_{n=0}^{\infty}\sum_{\alpha}\frac{(-i\lambda\hbar)^n}{n!}{}_{\mathrm{G}}\langle 0, t|[\int \hat{H}_{\mathrm{P}}(t)dt, \hat{\phi}^2_{\alpha}(t)]_{(n)}|0, t\rangle_{\mathrm{G}}e^{i\mathbf{k}\cdot\mathbf{x}} \\ &\approx \int\frac{d^3\mathbf{k}}{(2\pi)^3}\hbar\varphi^*_{\mathbf{k}}\varphi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}} + \frac{(\lambda\hbar)^2}{2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\hbar}{12}\Bigg[\int\varphi^4_{\mathbf{k}}dt\Bigg(\varphi^*_{\mathbf{k}}\varphi_{\mathbf{k}}\int\varphi^{*4}_{\mathbf{k}}dt \\ &\quad - \varphi^{*2}_{\mathbf{k}}\int\varphi^{*3}_{\mathbf{k}}\varphi_{\mathbf{k}}dt\Bigg)\int\varphi^{*4}_{\mathbf{k}}dt\Bigg(\varphi^*_{\mathbf{k}}\varphi_{\mathbf{k}}\int\varphi^4_{\mathbf{k}}dt - \varphi^2_{\mathbf{k}}\int\varphi^*_{\mathbf{k}}\varphi^3_{\mathbf{k}}dt\Bigg)\Bigg]e^{i\mathbf{k}\cdot\mathbf{x}},\end{aligned} \tag{25}$$

where $[\hat{O}, \hat{B}]_{(n)} = [\hat{O}, [\hat{O}, \hat{B}]_{(n-1)}]$ with $[\hat{O}, \hat{B}]_0 = \hat{B}$. Note that all the terms linear in $(\lambda\hbar)$ vanish and at order $(\lambda\hbar)^2$ the mode-mixing terms contribute quadratic terms of $\varphi_{\mathbf{k}}$ or $\varphi^*_{\mathbf{k}}$ to the scale dependent part, which are negligible compared to the quartic terms in Eq. (25).

Finally, after inserting the solution (23) into Eq. (25) and explicitly doing the integral, we obtain the two-point correlation function of the form

$$G_0(0, t)\frac{\sin\Big(\sqrt{\frac{m_f}{2t}}r\Big)}{\sqrt{\frac{m_f}{2t}}r}\exp\Big[-\frac{m_f r^2}{8t}\Big] + \frac{(\lambda\hbar)^2}{2}G_1(0, t)\frac{\sin\Big(\sqrt{\frac{m_f}{2\cdot 3t}}r\Big)}{\sqrt{\frac{m_f}{2\cdot 3t}}r}\exp\Big[-\frac{m_f r^2}{8\cdot 3t}\Big], \tag{26}$$

where $G(0, t)$'s are slowly varying functions of t. The leading and sub-leading terms of the second integral of Eq. (25) cancel each other and the third-leading term is the second term of Eq. (26). The first and the second terms of Eq. (26) are the Gaussian and the non-Gaussian contribution, respectively, to the correlation function. Each term oscillates and is

bounded by an envelope determined by exponentially decreasing functions. The first term, the Gaussian term, has a larger initial amplitude but decays more rapidly than the second term, the non-Gaussian term. Therefore, immediately after the phase transition, the first term determines the domain size, which obeys the well-known Cahn-Allen scaling relation [8–12]

$$\xi_{\rm G}(t) = \sqrt{\frac{8t}{m_f}}. \tag{27}$$

However, as the phase transition continues, the second term begins to dominate over the first term and leads to the non-Gaussian scaling relation

$$\xi_{\rm NG}(t) = \sqrt{\frac{8 \cdot 3t}{m_f}} = \sqrt{3}\xi_{\rm G}. \tag{28}$$

A few comments are in order. First, as the time for spinodal instability goes on, higher order non-Gaussian terms begin to grow comparable to and eventually become larger than the Gaussian and lower order terms. A series of transitions of scale relation may be expected from the Gaussian one to the non-Gaussian ones of higher orders. In fact it cannot happen in a realistic scenario because of the sampling of the mean field around the true vacuum and the de-excitation of excited states at order $(\lambda\hbar)^2$ or higher. The sampling of the mean field $\varphi_{\mathbf{k}}(t)$ around the true vacuum after crossing the spinodal line stops exponential growth and limits the duration of spinodal instability. Also there will be equally excitation and de-exitation, thus making virtually impossible for a system to achieve a state with inverted population, the very high excitation having the largest population. In reality there will be an increased occupation of excited states near the Gaussian state and the very highly excited states will have a small occupation probability. Second, though we focus on the real scalar field, the result above can be readily extended to a scalar field with $O(N)$ symmetry before the phase transition. The two-point function of a complex scalar field is essentially the same as that of the real scalar field [12]. We thus anticipate that other topological defects, such as monopoles, strings, and vortices, formed from the scalar field with the appropriate internal $O(N)$ symmetry may have the same correlation length as domains. The non-Gaussian effects thus may decrease the defect density.

Finally we conclude with the physical implication of non-Gaussian effects on domain growth, DCC and the density of topological defects. The RHIC at Brookhaven and the LHC at CERN in the near future will produce heavy ion beams at very high energies that will lead to deconfinement of quarks present inside individual nucleons. It is anticipated that a quark-gluon plasma will be formed and approach equilibrium over a time $\approx 10^{-23}s$ and then will evolve rapidly far from equilibrium through a chiral phase transition. It has been suggested that the chiral phase transition will be accompanied by the formation of DCC [6]. From a theoretical point of view it was argued that the linear σ model may be a reasonable model, that includes some of the symmetry properties such as chiral symmetry of QCD, to establish the extent of DCC. Rajagopal and Wilczek have advocated that the exponential growth of soft modes of the linear σ model may lead to a large correlation and hence a large domain of condensate [6]. This mechanism has similarities to the spinodal decomposition in QFT. The numerical simulation based on the Hartree-Fock approximation, however, results in smaller ($1 \sim 2$ fm) DCC regions than the anticipated (> 10 fm) DCC regions [19].

On the other hand, we find that the non-Gaussian effects increase the domain volume by an order of magnitude. This implies that a chiral phase transition may result in the formation of a large DCC domain, the decay of which eventually leads to a profuse production of coherent pions. However any extraordinary observation should be investigated carefully to find if there is a linkage to the DCC. The present study of a real scalar field serves only as a simple model that suggests the role of non-Gaussian effects on domains, and implies that the density of topological defects may be small. A realistic study of DCC formation during the chiral phase transition would require working with the linear σ model. It may be anticipated that long wavelength modes will play a similar role to enhance not only the size of domains in quark-gluon plasma, condensed matter and the early Universe but also the density perturbation from the second order phase transitions in inflation scenarios. These studies are in progress.

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